Transformation matrices used

Translation:

$$T(t_x, t_y, t_z) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation around x and y axis:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthogonal projection:

$$OP(l, r, b, t, n, f) = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{l+r}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

View at transform

The transpose operator is assuming column vectors.

$$lookAt(\overrightarrow{eye}, \overrightarrow{at}, \overrightarrow{up}) = \begin{bmatrix} \overrightarrow{xaxis}^T & -\overrightarrow{xaxis} \cdot \overrightarrow{eye} \\ \overrightarrow{yaxis}^T & -\overrightarrow{yaxis} \cdot \overrightarrow{eye} \\ \overrightarrow{zaxis}^T & -\overrightarrow{zaxis} \cdot \overrightarrow{eye} \end{bmatrix}$$

where;

$$\overrightarrow{zaxis} = \frac{\overrightarrow{at} - \overrightarrow{eye}}{|\overrightarrow{at} - \overrightarrow{eye}|}$$

$$\overrightarrow{xaxis} = \frac{\overrightarrow{zaxis} \times \overrightarrow{up}}{|\overrightarrow{zaxis} \times \overrightarrow{up}|}$$

$$\overrightarrow{yaxis} = \overrightarrow{xaxis} \times \overrightarrow{zaxis}$$

Perspective projection:

Fov is assuming degrees and not radians.

$$PP(fov, ar, f, n) = \begin{bmatrix} \frac{1}{\tan\left(\frac{fov}{2}\right) \cdot ar} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan\left(\frac{fov}{2}\right)} & 0 & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & -\frac{2f \cdot n}{f-n} \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Application of transform matrices

Part 1 – Isometric cube:

$$\vec{v}' = OP(-1,1,-1,1,0.01,1000) \cdot lookAt\left(\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) \cdot T(0.5,0.5,0.5) \cdot \vec{v}$$

Part 2 – Perspective cubes:

Cube 1 – 1-point perspective:

$$\vec{v}' = PP(45, 1, 0.01, 1000) \cdot lookAt\left(\begin{bmatrix}0\\0\\-7\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) \cdot T(2,-1.5,0) \cdot \vec{v}$$

Cube 2 – 2-point perspective:

$$\vec{v}' = PP(45, 1, 0.01, 1000) \cdot lookAt\left(\begin{bmatrix}0\\0\\-7\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) \cdot T(0,-1.5,0) \cdot R_y(40) \cdot \vec{v}$$

Cube 3 – 3-point perspective:

$$\vec{v}' = PP(45, 1, 0.01, 1000) \cdot lookAt\left(\begin{bmatrix}0\\0\\-7\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right) \cdot T(0,-1.5,0) \cdot R_x(40) \cdot R_y(40) \cdot \vec{v}$$